THREE-DIMENSIONAL TEMPERATURE FIELD OF A DISCRETELY GROWING SEMIINFINITE COLUMN WITH HEAT RELEASE

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An analytic solution is obtained for the problem of the three-dimensional temperature field of a semiinfinite column whose height varies discretely with time. Heat release takes place in the growing part of the column. Heat is transferred from the column to the medium by convection.

Problems of this kind are encountered in investigating the temperature fields of massive hydraulic engineering structures during the construction period when they are divided into blocks for concrete pouring purposes.

This paper is concerned with the temperature field of a semiinfinite column with variable initial temperature. At a certain point the height of the column begins to increase as a result of the addition of discrete blocks. The blocks are added instantaneously, each block being characterized by its own constant initial temperature. Subsequently, heat is released in the block at a rate that depends exponentially on time. The intervals at which blocks are added may vary. The temperature of the medium and the heat transfer coefficients at the horizontal and vertical surfaces are also different. The thermophysical characteristics of the column are constant.

In this formulation the problem adequately reflects the thermal conditions associated with the pouring of a mass of concrete whose base is an "old" concrete column and is also reasonably close to the thermal conditions for concrete poured over a rock foundation.

In the given stage of growth let the column consist of a base and $\bar{n}$ blocks.

We locate the coordinate origin at the upper horizontal surface of the last $k = \bar{n}$ block and direct the OZ axis into the column. The directions of the OX and OY axes are as usual. Then the problem can be formulated mathematically as follows.

The system of differential equations is written

$$\frac{\partial T_0}{\partial \tau^{(\bar{n})}} = a\nabla^2 T_0 \quad \left(\sum_{j=1}^{\bar{n}} R_j < z_{\bar{n}} < \infty,\right.$$

$$\left. 0 < x < L, \quad 0 < y < D\right),$$

$$\frac{\partial T_k}{\partial \tau^{(\bar{n})}} = a\nabla^2 T_k + q_0 \exp[-mt_k],$$

$$\sum_{j=k+1}^{\bar{n}} R_j < z_{\bar{n}} < \sum_{j=k}^{\bar{n}} R_j$$

$$(k = 1,\ 2,\ \ldots,\ \bar{n}). \tag{1}$$

The initial conditions are

$$T_0(z_{\bar{n}},\ x,\ y,\ 0) = T^{(ck)} + \Phi_0^{\bar{n}-1}(z_{\bar{n}},\ x,\ y,\ \tau_{\bar{n}}),$$

$$T_l(z_{\bar{n}},\ x,\ y,\ 0) = \psi_{\bar{n}} + \Phi_c^{\bar{n}-1}(z_{\bar{n}},\ x,\ y,\ \tau_{\bar{n}})$$

$$(l = 1,\ 2, \ldots,\ \bar{n} - 1),$$

$$T_{\bar{n}}(z_{\bar{n}},\ x,\ y,\ 0) = T^{(\bar{n})}. \tag{2}$$

The boundary conditions at the surfaces of the growing parts of the column are

$$\frac{\partial T_{\bar{n}}(0,\ x,\ y,\ \tau^{(\bar{n})})}{\partial z_{\bar{n}}} = -h_{z(\bar{n})}\left[\varphi_{\bar{n}} - T_{\bar{n}}(0,\ x,\ y,\ \tau^{(\bar{n})})\right],$$

$$\frac{\partial T_k(z_{\bar{n}},\ L,\ y,\ \tau^{(\bar{n})})}{\partial x} = h_{x(\bar{n})}\left[\psi_{\bar{n}} - T_k(z_{\bar{n}},\ L,\ y,\ \tau^{(\bar{n})})\right],$$

$$\frac{\partial T_k(z_{\bar{n}},\ 0,\ y,\ \tau^{(\bar{n})})}{\partial x} = 0,$$

$$\frac{\partial T_k(z_{\bar{n}},\ x,\ D,\ \tau^{(\bar{n})})}{\partial y} = h_{y(\bar{n})}\left[\psi_{\bar{n}} - T_k(z_{\bar{n}},\ x,\ D,\ \tau^{(\bar{n})})\right],$$

$$\frac{\partial T_k(z_{\bar{n}},\ x,\ 0,\ \tau^{(\bar{n})})}{\partial y} = 0, \tag{3}$$

and at the surfaces of the base

$$\frac{\partial T_0(\infty,\ x,\ y,\ \tau^{(\bar{n})})}{\partial z_{\bar{n}}} = 0;$$

$$\frac{\partial T_0(z_{\bar{n}},\ L,\ y,\ \tau^{(\bar{n})})}{\partial x} = h_{x(\bar{n})}\left[T^{(ck)} - T_0(z_{\bar{n}},\ L,\ y,\ \tau^{(\bar{n})})\right],$$

$$\frac{\partial T_0(z_{\bar{n}},\ 0,\ y,\ \tau^{(\bar{n})})}{\partial x} = 0;$$

$$\frac{\partial T_0(z_{\bar{n}},\ x,\ D,\ \tau^{(\bar{n})})}{\partial y} = h_{y(\bar{n})}\left[T^{(ck)} - T_0(z_{\bar{n}},\ x,\ D,\ \tau^{(\bar{n})})\right],$$

$$\frac{\partial T_0(z_{\bar{n}},\ x,\ 0,\ \tau^{(\bar{n})})}{\partial y} = 0. \tag{4}$$

The continuity conditions at the interblock boundaries are

$$T_k\left(\sum_{j=k}^{\bar{n}} R_j,\ x,\ y,\ \tau^{(\bar{n})}\right) = \tag{5}$$

$$= T_{k-1}\left(\sum_{j=k}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right),$$

$$\frac{\partial T_k\left(\sum_{j=k}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right)}{\partial z_{\bar n}} = \frac{\partial T_{k-1}\left(\sum_{j=k}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right)}{\partial z_{\bar n}} \quad (5)\ \text{(cont'd)}$$

and at the block-base boundary

$$T_1\left(\sum_{j=1}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right) = T_0\left(\sum_{j=1}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right),$$

$$\frac{\partial T_1\left(\sum_{j=1}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right)}{\partial z_{\bar n}} = \frac{\partial T_0\left(\sum_{j=1}^{\bar n} R_j,\ x,\ y,\ \tau^{(\bar n)}\right)}{\partial z_{\bar n}}. \quad (6)$$

The solution of this problem can be obtained by means of double cosine transformations, Laplace transformations, and the Green's function [1, 2, 4].

The result is

$$T_0(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = T^{(ck)} + \Phi_0^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}),$$

$$T_k(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = \psi_{\bar n} + \Phi_c^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)})$$

$$(k = 1,\ 2,\ \ldots,\ \bar n), \quad (7)$$

where

$$\Phi_i^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = F^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) + f_i(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) + \frac{4}{LD}\sum_{s=1}^{\infty}\sum_{r=1}^{\infty}\frac{(\mathrm{Bi}_x+\mu_s^2)(\mathrm{Bi}_y+\varkappa_r^2)}{(\mathrm{Bi}_x^2+\mathrm{Bi}_x+\mu_s^2)(\mathrm{Bi}_y^2+\mathrm{Bi}_y+\varkappa_r^2)}\times$$

$$\times \exp\left[-Ka\tau^{(\bar n)}\right]\times \int_{R_{\bar n}}^{\infty}\int_0^L\int_0^D \left[\Phi^{(\bar n-1)}(\xi,\ x,\ y,\ \tau_{\bar n})\right]\times \cos\mu_s\frac{x}{L}\cos\varkappa_r\frac{y}{D}\times$$

$$\times\left\{\frac{1}{2\sqrt{\pi a\tau^{(\bar n)}}}\left[\exp\left[-\frac{(z_{\bar n}-\xi)^2}{4a\tau^{(\bar n)}}\right] + \exp\left[-\frac{(z_{\bar n}+\xi)^2}{4a\tau^{(\bar n)}}\right]\right] - h_{z(\bar n)}\exp\left[h_{z(\bar n)}^2 a\tau^{(\bar n)} + h_{z(\bar n)}(z_{\bar n}+\xi)\right]\times\right.$$

$$\left.\times\operatorname{erfc}\left(\frac{z_{\bar n}+\xi}{2\sqrt{a\tau^{(\bar n)}}} + h_{z(\bar n)}\sqrt{a\tau^{(\bar n)}}\right)\right\}d\xi\,dx\,dy \quad (i = 0,\ c),$$

$$\left[\Phi^{(\bar n-1)}(\xi,\ x,\ y,\ \tau_{\bar n})\right] = \begin{cases}\Phi_0^{(\bar n-1)}(\xi,\ x,\ y,\ \tau_{\bar n}) & \left(\sum_{j=1}^{\bar n} R_j < z_{\bar n} < \infty\right)\\ \Phi_c^{(\bar n-1)}(\xi,\ x,\ y,\ \tau_{\bar n}) & \left(R_{\bar n} < z_{\bar n} < \sum_{j=1}^{\bar n} R_j\right),\end{cases}$$

$$K = \frac{\mu_s^2}{L^2} + \frac{\varkappa_r^2}{D^2}. \quad (8)$$

The functions are found from the following expressions:

$$f_0^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = \sum_{s=1}^{\infty}\sum_{r=1}^{\infty} A_s B_r\cos\mu_s\frac{x}{L}\cos\varkappa_r\frac{y}{D}\times\left[T^{(n)} - \psi_{\bar n} + \frac{q_0}{m-aK}\right]\times$$

$$\times\exp\left[-Ka\tau^{(\bar n)}\right] - \Theta(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}), \quad (9)$$

$$f_c^{(\bar n)}(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = \sum_{s=1}^{\infty}\sum_{r=1}^{\infty} A_s B_r\cos\mu_s\frac{x}{L}\cos\varkappa_r\frac{y}{D}\times\left[(T^{(\bar n)}-\psi_{\bar n})\exp\left[-Ka\tau^{(\bar n)}\right] + \frac{q_0}{m-aK}\left(\exp\left[-Ka\tau^{(\bar n)}\right] - \exp\left[-m\left(\sum_{i=2}^{\bar n}\tau_j + \tau^{(\bar n)}\right)\right]\right)\right] +$$

$$+\Theta(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}), \quad (10)$$

$$\Theta(z_{\bar n},\ x,\ y,\ \tau^{(\bar n)}) = \frac{1}{4}(T^{(ck)}-\psi_{\bar n})\times\sum_{s=1}^{\infty}\sum_{r=1}^{\infty}A_s B_r\cos\mu_s\frac{x}{L}\cos\varkappa_r\frac{y}{D}\times$$

$$\times\left\{\exp\left[-\sqrt{K}\left(\sum_{j=1}^{\bar n}R_j - z_{\bar n}\right)\right]\times\operatorname{erfc}\left(\frac{\sum_{j=1}^{\bar n}R_j - z_{\bar n}}{2\sqrt{a\tau^{(\bar n)}}} - \sqrt{Ka\tau^{(\bar n)}}\right) + \exp\left[\sqrt{K}\left(\sum_{j=1}^{\bar n}R_j - z_{\bar n}\right)\right]\times\operatorname{erfc}\left(\frac{\sum_{j=1}^{\bar n}R_j - z_{\bar n}}{2\sqrt{a\tau^{(\bar n)}}} + \sqrt{Ka\tau^{(\bar n)}}\right)\right\}, \quad (11)$$

Values of the Coefficients $a_{\rho i}$, $b_{\rho i}$, $d_{\rho i}$, $g_{\rho i}$, c_ρ

a_{1i}	a_{2i}	a_{3i}	a_{4i}
$-\frac{q_0 \exp\left[-m\sum_{j=i+2}^{\bar n}\tau_j\right]}{2(m-aK)}(1-\exp[-m\tau_{i+1}])$, $i\neq\bar n$, $\psi_{\bar n}-T^{(\bar n)}+\frac{q_0}{m-aK}$, $i=\bar n$,	$-a_{1i}$, $i\neq\bar n$, 0, $i=\bar n$,	$\frac{q_0\exp\left[-m\sum_{j=2}^{\bar n}\tau_j\right]}{2(m-aK)}$, $i=1$, 0, $i\neq 1$,	$-a_{3i}$
b_{1i}	b_{2i}	b_{3i}	b_{4i}
$-\frac{q_0\exp\left[-m\sum_{j=i+2}^{\bar n}\tau_j\right]}{(h^2_{z(\bar n)}a+m-aK)}(1-\exp[-m\tau_{i+1}])$, $i\neq\bar n$, $T^{(\bar n)}-\varphi_{\bar n}+\frac{q_0}{h^2_{z(\bar n)}a+m-aK}-$ $-\frac{(\varphi_{\bar n}-\psi_{\bar n})h^2_{z(\bar n)}}{(h^2_{z(\bar n)}-K)}$, $i=\bar n$,	$\frac{(T^{(ck)}-\psi_{\bar n})h^2_{z(\bar n)}}{(h^2_{z(\bar n)}-K)}-$ $-\frac{q_0\exp\left[-m\sum_{j=2}^{\bar n}\tau_j\right]}{(h^2_{z(\bar n)}a+m-aK)}$, $i=1$, 0, $i\neq 1$,	0	0
d_{1i}	d_{2i}	d_{3i}	d_{4i}
$-\frac{h_{z(\bar n)}\sqrt{a}\,q_0\exp\left[-m\sum_{j=i+2}\tau_j\right]}{\sqrt{\pi}(h^2_{z(\bar n)}a+m-aK)}\times$ $\times(1-\exp[-m\tau_{i+1}])$, $i\neq\bar n$, $\frac{h_{z(\bar n)}\sqrt{a}\,q_0}{\sqrt{\pi}(h^2_{z(\bar n)}a+m-aK)}$, $i=\bar n$	$-\frac{h^2_{z(\bar n)}aq_0\exp\left[-m\sum_{j=2}^{\bar n}\tau_j\right]}{\sqrt{\pi}(h^2_{z(\bar n)}a+m-aK)}$, $i=1$, 0, $i\neq 1$,	0	0
g_{1i}	g_{2i}	g_{3i}	g_{4i}
$-\frac{(h^2_{z(\bar n)}a-m+aK)q_0\exp\left[-m\sum_{j=i+2}^{\bar n}\tau_j\right]}{\sqrt{\pi}(m-aK)(h^2_{z(\bar n)}a+m-aK)}\times$ $\times(1-\exp[-m\tau_{i+1}])$, $i\neq\bar n$ $\frac{2h^2_{z(\bar n)}aq_0}{\sqrt{\pi}(m-aK)(h^2_{z(\bar n)}a+m-aK)}$, $i=\bar n$	$\frac{q_0\exp\left[-m\sum_{j=i+2}^{\bar n}\tau_j\right]}{\sqrt{\pi}(m-aK)}\times$ $\times(1-\exp[-m\tau_{i+1}])$, $i\neq\bar n$ 0, $i=\bar n$	$-\{(h^2_{z(\bar n)}a-m+aK)q_0\times$ $\times\exp\left[-m\sum_{j=2}^{\bar n}\tau_j\right]\}\times$ $\times\{\sqrt{\pi}(m-aK)\times$ $\times(h^2_{z(\bar n)}a+m-aK)\}^{-1}$, $i=1$, 0, $i\neq 1$	$-\frac{q_0\exp\left[-m\sum_{j=2}^{\bar n}\tau_j\right]}{\sqrt{\pi}(m-aK)}$ $i=1$ 0, $i\neq 1$
c_1	c_2	c_3	c_4
$-\frac{(T^{(ck)}-\psi_{\bar n})(h_{z(\bar n)}+\sqrt{K})}{4(h_{z(\bar n)}-\sqrt{K})}$	$-\frac{(T^{(ck)}-\psi_{\bar n})(h_{z(\bar n)}-\sqrt{K})}{4(h_{z(\bar n)}+\sqrt{K})}$,	$-\frac{(\psi_{\bar n}-\varphi_{\bar n})h_{z(\bar n)}}{2(h_{z(\bar n)}-\sqrt{K})}$,	$-\frac{(\psi_{\bar n}-\varphi_{\bar n})h_{z(\bar n)}}{2(h_{z(\bar n)}+\sqrt{K})}$

$$F^{(\bar n)}(z_{\bar n}, x, y, \tau^{(\bar n)}) =$$

$$= \sum_{s=1}^{\infty}\sum_{r=1}^{\infty} A_s B_r \cos\mu_s \frac{x}{L} \cos\varkappa_r \frac{y}{D} \times$$

$$\times \left\{ \sum_{\rho=1}^{4} \left\{ \exp[-Ka\tau^{(\bar n)}] \times \right.\right.$$

$$\times \left[\sum_{i=1}^{\bar n} \left(a_{\rho i} \operatorname{erfc}\left[\frac{N_\delta(z_{\bar n})}{2\sqrt{a\tau^{(\bar n)}}}\right] + \right.\right.$$

$$+ b_{\rho i} \exp\left[(h^2_{z(\bar n)} + K) a\tau^{(\bar n)} + h_{z(\bar n)} N_\gamma(z_{\bar n})\right] \times$$

$$\times \operatorname{erfc}\left[h_{z(\bar n)}\sqrt{a\tau^{(\bar n)}} + \frac{N_\gamma(z_{\bar n})}{2\sqrt{a\tau^{(\bar n)}}}\right] +$$

$$+ d_{\rho i} I_{1/2}\left[m - aK, -\left(\frac{N_\gamma(z_{\bar n})}{2\sqrt{a}}\right)^2, \tau^{(\bar n)}\right] +$$

$$\left.\left. + g_{\rho i} I_{3/2}\left[m - aK, -\left(\frac{N_\delta(z_{\bar n})}{2\sqrt{a}}\right)^2, \tau^{(\bar n)}\right]\right)\right] +$$

$$+ c_\rho \exp\left[(-1)^{\rho-1}\sqrt{K}\left(z_{\bar n} + \delta_\rho \sum_{j=1}^{\bar n} R_j\right)\right] \times$$

$$\times \operatorname{erfc}\left[\frac{z_{\bar n} + \delta_\rho \sum_{j=1}^{\bar n} R_j}{2\sqrt{a\tau^{(\bar n)}}} + \right.$$

$$\left.\left.\left. + (-1)^{\rho-1}\sqrt{Ka\tau^{(\bar n)}}\right]\right\}\right\}, \tag{12}$$

$$N_\gamma(z_{\bar n}) = \sum_{j=1}^{\bar n} R_j + \gamma_\rho \sum_{j=1}^{i} R_j + z_{\bar n}, \tag{13}$$

$$N_\delta(z_{\bar n}) = \sum_{j=1}^{\bar n} R_j - \delta_\rho \sum_{j=1}^{i} R_j + (-1)^{\rho-1} z_{\bar n}, \tag{14}$$

$$A_s = (-1)^{s+1} \frac{2\mathrm{Bi}_x \sqrt{\mathrm{Bi}_x^2 + \mu_s^2}}{\mu_s(\mathrm{Bi}_x^2 + \mathrm{Bi}_x + \mu_s^2)}, \tag{15}$$

$$B_r = (-1)^{r+1} \frac{2\mathrm{Bi}_y \sqrt{\mathrm{Bi}_y^2 + \varkappa_r^2}}{\varkappa_r(\mathrm{Bi}_y^2 + \mathrm{Bi}_y + \varkappa_r^2)}, \tag{16}$$

where

$$\delta_\rho = \begin{cases} 1 & \text{at} \quad \rho \leqslant 2 \\ 0 & \text{at} \quad \rho > 2 \end{cases}; \tag{17}$$

$$\gamma_\rho = \begin{cases} 1 & \text{at} \quad \rho = 1 \\ 0 & \text{at} \quad \rho > 1 \end{cases}. \tag{18}$$

The algorithm proposed by the author in [3] is recommended for evaluating the integrals

$$I_{1/2}\left[m - aK, -\left(\frac{N}{2\sqrt{a}}\right)^2, \tau^{(\bar n)}\right] =$$

$$= \int_0^{\tau^{(\bar n)}} \frac{\exp[(m - aK)(\Theta - \tau^{(\bar n)}) - N^2/4a\Theta]}{\Theta^{1/2}} d\Theta \tag{19}$$

and

$$I_{3/2}\left[m - aK, -\left(\frac{N}{2\sqrt{a}}\right)^2, \tau^{(\bar n)}\right] = \frac{N}{2\sqrt{a}} \times$$

$$\times \int_0^{\tau^{(\bar n)}} \frac{\exp[(m - aK)(\Theta - \tau^{(\bar n)}) - N^2/4a\Theta]}{\Theta^{3/2}} d\Theta. \tag{20}$$

The same paper also gives tables of values of these integrals.

As usual,

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp[-x^2]\, dx =$$

$$= \frac{2}{\sqrt{\pi}} \int_x^\infty \exp[-x^2]\, dx.$$

The coefficients $a_{\rho i}$, $b_{\rho i}$, $d_{\rho i}$, $g_{\rho i}$, c_ρ are found from the expressions presented in the table.

In determining the function $\Phi^{(\bar n)}(z_{\bar n}, x, y, \tau^{(\bar n)})$ the evaluation of the integral in expression (8) is particularly important. The integrand contains the value $\Phi^{(\bar n-1)}$ for a column consisting of $(\bar n - 1)$ blocks at the moment when it is covered with the $\bar n$-th block.

We write

$$\int_{R_{-\bar n}}^{\infty} \int_0^L \int_0^D [\Phi^{(\bar n-1)}(\xi, x, y, \tau^{(\bar n)})] \times$$

$$\times \cos\mu_s \frac{x}{L} \cos\varkappa_r \frac{y}{D} \{\ \} d\xi\, dx dy =$$

$$= \int_{R_{-\bar n}}^{\sum_{j=1}^{\bar n}(R_j + R_{-j})} \int_0^L \int_0^D + \int_{\sum_{j=1}^{\bar n}(R_j + R_{-j})}^{\infty} \int_0^L \int_0^D \cos\mu_s \frac{x}{L} \times$$

$$\times \cos\varkappa_r \frac{y}{D} \{\ \} d\xi\, dx dy. \tag{21}$$

Here, R_{-j} is the height of an "imaginary" block, which in a certain sense is the reflection in the base of a real block of height R_j.

In the general case the height R_{-j} is a multiple of the height R_j.

Since at large $z_{\bar n}$ the function $\Phi^{(\bar n-1)}$ tends to zero, we can always choose a factor such that the second integral in (21) also tends to zero.

The evaluation of the first integral will be simplified if within the limits of each block, real and "imaginary," the function $\Phi^{(\bar n-1)}$ is approximated by a formula of the type

$$\Phi_k^{(\bar n-1)} = P_\alpha(z_{\bar n}) P_\beta(x) P_\gamma(y),$$

where P_α, P_β, P_γ are polynomials of degree α, β, γ.

The degree of the polynomial depends on the specified accuracy. As it is easy to show, with the above approximation all the computations reduce to calculations based on simple recurrence relations.

NOTATION

n are the maximum number of blocks in column; $\bar{n}$ are the number of blocks in the column at a particular stage of growth ($\bar{n} = 1, 2, \ldots, n$); k is the number of the block (counting from the base for which $k = 0$) ($k = 1, 2, \ldots, \bar{n}$); $2L \times 2D$ are the plan dimensions of column; R_k is the height of k-th block; T is the temperature; T_k (T_l) and T_0 are the temperatures of the k-th (l-th) block and base at any instant of time; $T^{(k)}$ is the temperature of the k-th block at the instant of pouring (constant); $T^{(ck)}$ is the constant component of the base temperature and also the temperature of the medium at vertical surfaces in the base region; $\varphi_{\bar{n}}$ and $\psi_{\bar{n}}$ are the temperatures of the medium at the horizontal and vertical surfaces in region of blocks during pouring of the next $\bar{n}$-th block; τ is the time; $\tau^{(\bar{n})}$ is the time from instant of pouring of the next $\bar{n}$-th block; $\tau_{\bar{n}}$ is the interval between pouring of the ($\bar{n} - 1$)-th block and its being covered with the $\bar{n}$-th block; t_k is the "lifetime" of the k-th block; x, y, z are coordinates; $z_{\bar{n}}$ is the coordinate z for the column consisting of $\bar{n}$ blocks and base; $h_{x(\bar{n})}$, $h_{y(\bar{n})}$, $h_{z(\bar{n})}$ are the relative heat transfer coefficients for column at a particular stage of growth; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z_{\bar{n}}^2$; $q_0 = q_0'/c\gamma$; $Bi_x = h_{x(\bar{n})}L$; $Bi_y = h_{y(\bar{n})}D$; μ_s is the root of the transcendental equation $\operatorname{ctg} \mu_s = \mu_s/Bi_x$; $\varkappa_r$ is the root of the transcendental equation $\operatorname{ctg} \varkappa_r = \varkappa_r/Bi_y$.

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